be computed as follows

$$M_{r\max} = 0.01\sqrt{Re_r} \quad \text{if } Re_r > 1$$

$$M_{r\max} = 0.01 \quad Re_r \quad \text{if } Re_r < 1 \tag{4}$$

The C_D method presented in Ref. 3 should be modified as follows to give a smooth transition between Eqs. (1) and (2)

$$\text{for } M_r \geq 0.1 \quad C_D = C_{D1} = C_{DC} + (C_{DFM} - C_{DC})\, e^{-ARe_r^N}$$

$$\text{for } M_r < M_{r\max} < 0.1 \quad C_D = C_{D2} = 24/Re_r(1 + 0.15\, Re_r^{\,0.687})$$

$$\text{for } M_{r\max} < M_r < 0.1 \quad C_D = \frac{C_{D1} - C_{D2}}{0.1 - M_{r\max}}\,(M_r - M_{r\max}) + C_{D2} \tag{5}$$

In summary, the sphere drag correlation presented by Henderson[1] and the present author[3] should provide similar prediction accuracy of C_D for $M_r > 1.75$; however, the present author's C_D method[3] gives better predictions of C_D for $M_r < 1.75$. However, the method originally presented in Ref. 3 should be modified for $M_r < 0.1$ as described herein.

References

[1]Henderson, C. B., "Drag Coefficients of Spheres in Continuum and Rarefied Flows," *AIAA Journal*, Vol. 14, June 1966, pp. 707-708.

[2]Bailey, A. B. and Hiatt, J., "Free-Flight Measurements of Sphere Drag at Subsonic, Transonic, Supersonic, and Hypersonic Speeds for Continuum, Transition, and Near-Free-Molecular Flow Conditions," Arnold Engineering Development Center, Tullahoma, Tenn., AEDC-TR-70-291, March 1971.

[3]Walsh, M. J., "Drag Coefficient Equations for Small Particles in High Speed Flows," *AIAA Journal*, Vol. 13, Nov. 1975, pp. 1526-1528.

[4]Walsh, M. J., "Influence of Particle Drag Coefficient on Particle Motion in High-Speed Flow with Typical Laser Velocimetry Applications," NASA TN D-8120, Feb. 1976.

[5]Zarin, N. A., "Measurement of Non-Continuum and Turbulence Effects on Subsonic Sphere Drag," NASA CR-1585, June 1970.

[6]Emmons, H. W., Ed., *Fundamentals of Gas Dynamics, Vol. III. High Speed Aerodynamics and Jet Propulsion*, Princeton University Press, Princeton, N.J., 1958, p. 689.

Reply by Author to M. J. Walsh

C. B. Henderson*

Atlantic Research Corporation, Alexandria, Va.,

THE stated purpose of Walsh's comment is to compare his correlation[1,2] of drag coefficients with one proposed by this author.[3] This comparison is accomplished within a restricted range of Mach numbers and Reynolds numbers. Figures 1 and 2 of Walsh's comment show that, at Reynolds numbers Re between 20 and 200 and at Mach numbers M of 0.5 and 1.25, his correlation is more accurate. This author is in agreement with this conclusion; a quantitative comparison of the two correlations with the experimental data of Bailey and Hiatt[4] shows that the maximum error within this range of conditions is reduced from 16% to 7% by use of the Walsh correlation. It should be pointed out, however, that this author's correlation was intended to be used over a much wider range of Reynolds numbers and Mach numbers, namely $0 < M < 6$ and $0 < Re < Re_{cr}$, where Re_{cr} is the Reynolds number at which turbulence produces a sudden reduction in the drag coefficient. The range of Reynolds numbers and Mach numbers covered by the two respective correlations is shown in Fig. 1.

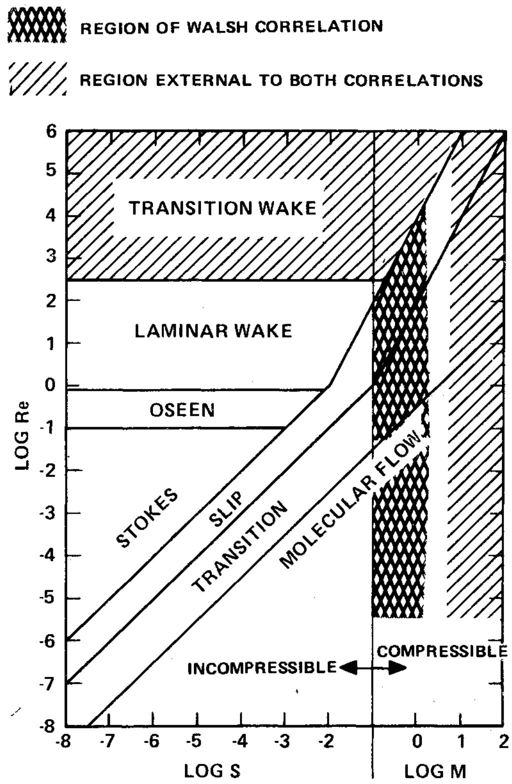

Fig. 1 Regions of applicability of C_D correlations.

It should be of interest to see how the two types of correlations compare in regions outside of the very narrow one in which Walsh made his comparison. Since Walsh[1,2] only gives correlating parameters for $0.1 < M < 2$, a quantitative comparison can be made only within that range. At higher Reynolds numbers, $200 < Re < 10^4$, the two correlations give the same maximum percentage deviation from the data of Bailey and Hiatt; specifically 7-8% at $0.1 < M < 0.5$ and 15% at $0.5 < M < 2.0$.

At Reynolds numbers below 20, there are no experimental data with which to compare; instead the limit of C_D in the molecular flow regime approached by both correlations at low Re can be investigated. Both Walsh and this author employ a theoretical equation to calculate C_D in the molecular flow regime. The equations used differ for two reasons. First, Walsh employs an equation for diffuse reflection only, while this author considers both diffuse and specular reflection; neglecting specular reflection results in C_D values which are higher by 3%. Second, Walsh's expression for diffuse reflection contains an error. The first term of Walsh's diffuse reflection equation (Eq. (9a), p. 4 of Ref. 2) is

$$(1 + 2s^2)\exp\left(\frac{-s^2/2}{\sqrt{\pi}s^3}\right)$$

This apparently contains a typographical error, since the numerical values given by Walsh are in agreement with the equation given in Walsh's basic reference (Schaaf and Chambre[5]), the first term of which is

$$\frac{(1 + 2s^2)}{\sqrt{\pi}s^3}\exp\,(-s^2/2)$$

Schaaf and Chambre[5] have a typographical error in their first term, however, as shown by comparison to the equation given by Stalder and Zurick,[6] the original authors of the equation.

Received March 17, 1977.

Index categories: Multiphase Flow; Nozzle and Channel Flow.

*Vice President. Associate Fellow AIAA.

The first term of Stalder and Zurick's equation is

$$\frac{(1+2s^2)}{\sqrt{\pi}s^3}\exp\ (-s^2)$$

At $s=0.8$, Walsh's C_D values are almost 10% higher than those calculated using Stalder and Zurick's equation for diffuse reflection. Thus, the set of 84 constants presented by Walsh for $0.1<M<2$ need to be revised to remove this error.

It is also of interest to consider Mach numbers greater than 1.75 and less than 0.1. Since both Walsh and this author use a Martino-type expression at $M>1.75$, both techniques should be capable of yielding the same degree of accuracy in this range. This is not true, however, at $M<0.1$. In Walsh's original presentation,[12] rarefaction effects are ignored at $M<0.1$; this leads to very large errors in C_D values. In his comment, Walsh attempts to correct this problem by proposing his Eq. (5). Inspection of the form of Eq. (5), however, shows that it must fail at low Reynolds number. Consider, for example, C_D at $M=Re=10^{-3}$. The continuum value of C_D is 2.4×10^4 and the free molecular value is 5×10^3. The term $(M-M_{\max})/(0.1-M_{\max})$ in Walsh's Eq. (5), however, is $\sim10^{-2}$. Thus, at low Re, $C_D\cong C_{D2}$; this is equivalent to ignoring rarefaction effects and can lead to errors in C_D as large as several orders of magnitude.

It is concluded that Walsh's correlation gives improved accuracy in a narrow range of Mach number and Reynolds number. Walsh's correlation is shown to fail at Mach numbers and Reynolds numbers less than 0.1. Finally, an error in Walsh's values for molecular flow C_D requires a revision in his empirical parameters.

References

[1] Walsh, M. J., "Drag Coefficient Equations for Small Particles in High Speed Flows," *AIAA Journal*, Vol. 13, Nov. 1975, pp. 1526-1528.

[2] Walsh, M. J., "Influence of Particle Drag Coefficient on Particle Motion in High-Speed Flow with Typical Laser Velocimetry Applications," NASA TND-8120, Feb. 1976.

[3] Henderson, C. B., "Drag Coefficients of Spheres in Continuum and Rarefied Flows," *AIAA Journal*, Vol. 14, June 1976, pp. 707-708.

[4] Bailey, A. B., and Hiatt, J., "Free-Flight Measurements of Sphere Drag at Subsonic, Transonic, and Supersonic Speeds for Continuum, Transition, and Near-Molecular Flows," Arnold Engineering Development Center, Arnold Air Force Station, Tenn., TR-70-291, March 1971.

[5] Schaaf, S. A., and Chambre, P. L., "Flow of Rarefied Gases," Emmons, H. W., Ed., *Fundamentals of Gas Dynamics*, Vol. III, High Speed Aerodynamics and Jet Propulsion," Princeton University Press, Princeton, N.J., 1958, p. 704.

[6] Stalder, J. and Zurick, V., "Theoretical Characteristics of Bodies in a Free Molecule Flow Field," NASA TN 2423, 1951.

Comment on "Experimental Investigation of the Boundary Layer on a Rotating Cylinder"

E.E. Covert*
MIT, Cambridge, Mass.

In Ref. 1, Morton et al. presented an excellent review of their experimental data on the characteristics of boundary layers on rotating cylinders. These data were reduced to the form of the distribution of displacement thickness and compared with predictions based upon J.C. Martin's theory (Ref. 2, herein; note this is Ref. 7 of Morton et al.). The purpose of these remarks is to clarify application of Martin's theoretical results to Morton's expansion of the three-dimensional boundary layer equation, including second order. The expansion contains a primary implicit assumption that the boundary layer thickness δ is much less than the radius of the cylinder r. For the data presented in Ref. 1, $0.07\le\delta/r\le0.17$, so the implicit assumption is satisfied, more or less.

Received Feb. 1, 1976.

Index category: Boundary-Layer Stability and Transition.

*Professor of Aeronautics and Astronautics. Fellow AIAA.

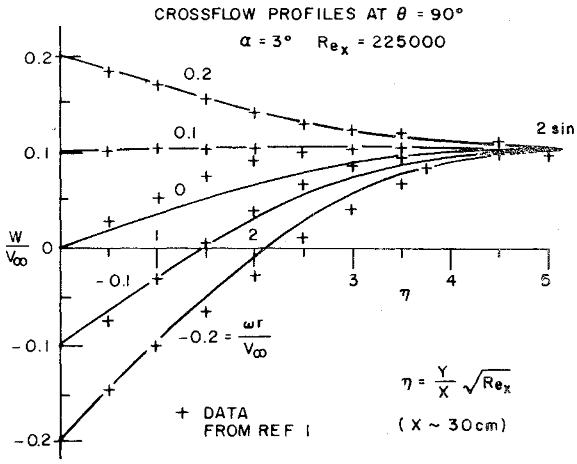

Fig. 1 Azimuthal velocity distribution for several spin rates.

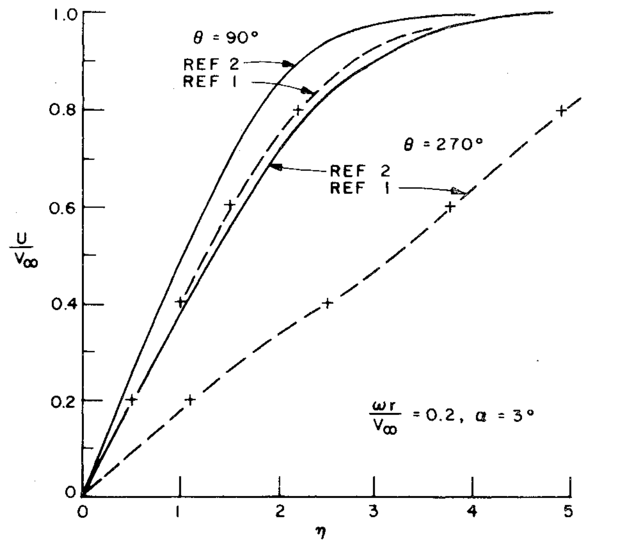

Fig. 2 Longitudinal velocity distribution through the boundary layer.

As Martin points out, his expansion scheme involves a number of dimensionless groups. The group of importance to this discussion is the product of the angle of attack α and the axial position measured in radii $(\alpha x/r)$. Its size is shown for several lengthwise positions, along the cylinder; for $x(cm)=$ 20, 30, 40, and 50; $(\alpha x/r)=0.37$, 0.55, 0.73, and 0.92; respectively. Assuming the series expansion in $(\alpha x/r)$ is convergent, the rate of convergence is likely to be slow for the values of x just listed.

The way in which Martin's approximate solution departs from the data for larger values of $(\alpha x/r)$ is of importance. Figure 1 contains a plot of Martin's solution for the azimuthal velocity distribution for the conditions given by Morton et al. The points have been determined from Fig. 4 of Ref. 1. Except for the zero spin case, the agreement between calculation and experiment is remarkably good. If the data in Fig. 3 are for the same conditions as Fig. 4 of Ref. 1, the measured longitudinal velocity profiles can be compared with the calculated profiles. Theoretically, the spin should have no